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Stability of Uncertain Networked Control Systems

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Abstract

When time-delay and data packet dropout are bounded, the networked control systems (NCSs) can be modeled as a class of systems with time-varying input delay. Based on Lyapunov-Krasovskii theorem, sufficient conditions in terms of linear matrix inequality (LMI) for the asymptotic stability of the nominal systems and uncertain systems are derived by the free weighting matrix approach. An integral term of Lyapunov functional derivative is reasonably transformed by Moon's inequality and Newton-Leibniz formula in this paper, while it is commonly ignored in most of the previous literatures, so less conservative results are obtained. The effectiveness of the criteria and algorithm are proved through numerical examples. The simulation results show that the system conservativeness is reduced significantly.

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Keywords: Networked control systems; Parameter uncertainty; Time-varying delay; Lyapunov functional; Linear matrix inequality

1. Introduction

Feedback control systems wherein the control loops are closed through networks like Internet are called networked control systems (NCSs). Compared to the point-to-point architecture systems, the primary advantages of the NCSs are low cost, reduced weight, simple installation and maintenance, high reliability [1]. Since the late 1970s, the NCSs have been applied in many complex systems such as remote medical treatment, transportation system, aerospace, manufacturing process and defense industry, etc. [2].

However, due to the network bandwidth limit and node competition, there exist transmission delay and data packet dropout in the NCSs, which may degrade the performance of control system and even destabilize the system [1]. The time-delay and data packet dropout can be constant or time-varying and their properties depend on the communication protocols and topology of the networks.

Nowadays, research on the NCSs is one of the hot spot of automation and network area. The scholars have made significant progress on this aspect and most of the previous works use discrete-time methods,

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such as random optimal control theory [3], switching system theory [4], predictive control theory [5], fuzzy control theory [6], asynchronous dynamic system theory [1], etc.

In literature [7], the sampled system was firstly modeled as a continuous-time one. Then the sufficient condition for stability in terms of linear matrix inequality (LMI) was derived. The main difference between the NCSs and the sampled systems is time-delay and data packet dropout, so the NCSs can also be modeled as continuous-time systems. Through proper transformations, the same method was used in the NCSs in literature [8-9], where less conservative results were obtained.

Time-delay systems have been studied for more than half a century and a large number of results and methods have been obtained, such as descriptor method [7], reduction method [10], free weighting matrix method [11], etc. Essentially, the NCSs are also a kind of systems with time-varying input delay, so the conclusions and methods of time-delay systems can also be used in the NCSs. In literature [8-9], the NCSs with random transmission delay and packet dropout were modeled as continuous-time systems with time-varying input delay, and both stability analysis and controller design criteria were presented based on the free weighting matrix approach. But it was pointed out in literature [11] that the results were conservative because an integral term of Lyapunov functional derivative was ignored in [8-9] and other similar literatures. Though literature [11] have not ignored this term, the method used to cope it is too simple and there is still room for improving.

Since only stable systems can work properly under disturbance, the stability is the most fundamental problem in the design of control systems. In this paper we use the free weighting matrix approach to analyze the stability of the NCSs. At first, the NCSs with time-delay and data packet dropout are modeled as continuous-time systems with time-varying input delay. Then the sufficient conditions in terms of LMI for asymptotic stability of the nominal systems and uncertain ones are derived respectively and the conservative problems existed in the [8-9, 11] and other similar literatures are overcome. Finally two examples are given to demonstrate the feasibility of the proposed criteria and algorithm.

2. Problem statement

A typical NCSs setup is shown in Fig. 1, where all control information is transmitted via network. There are three kinds of delays in the NCSs. In Fig. 1, τ_{sc}, τ_{ca} and τ_c represent the sensor-to-controller delay, controller-to-actuator delay and computational delay, respectively. Without loss of generality, the computational delay can be ignored because it is small compared to the other two kinds of delays. The number of consecutive data packet dropout between the sensor and controller is denoted as d_{sc} and the number of consecutive dropout between the controller and actuator is denoted as:.

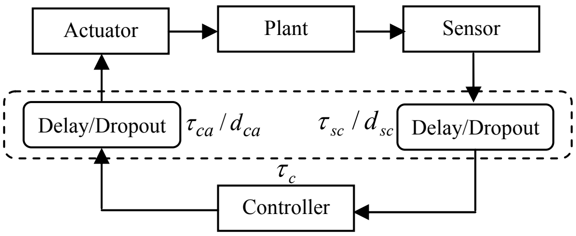

Fig. 1 A typical setup of the networked control systems

Consider the uncertain networked control systems described by the following state equation:

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \boldsymbol{\Delta A}(t))\boldsymbol{x}(t) + (\boldsymbol{B} + \boldsymbol{\Delta B}(t))\boldsymbol{u}(t), \tag{1}$$

where $\boldsymbol{x}(t) \in \mathrm{R}^n$ is the state vector, $\boldsymbol{u}(t) \in \mathrm{R}^m$ is the control input vector; $\boldsymbol{A} \in \mathrm{R}^{n\times n}$ and $\boldsymbol{B} \in \mathrm{R}^{n\times m}$ are constant matrices with appropriate dimensions; $\boldsymbol{\Delta A}(t)$ and $\boldsymbol{\Delta B}(t)$ are continuous real matrix functions representing the uncertainties in the system matrices $\boldsymbol{A}$ and $\boldsymbol{B}$. In this paper, we assume that the uncertainties can be described by

$$\begin{bmatrix}\boldsymbol{\Delta A}(t) & \boldsymbol{\Delta B}(t)\end{bmatrix} = \boldsymbol{D}\boldsymbol{F}(t)\begin{bmatrix}\boldsymbol{E}_a & \boldsymbol{E}_b\end{bmatrix}, \tag{2}$$

where $\boldsymbol{D}$, $\boldsymbol{E_a}$ and $\boldsymbol{E_b}$ are known constant matrices with appropriate dimensions, $\boldsymbol{F}(t)$ is an uncertain matrix with Lebesgue measurable elements bounded by

$$\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t) \le \boldsymbol{I} \quad \forall t . \tag{3}$$

The following assumptions are needed for the considered NCSs in this paper.

(i) The sensor node is time-driven with the fixed sampling period T, whereas the controller and actuator nodes are event-driven. All control data is transmitted with a single packet.

(ii) The controller uses the static state-feedback control law, so the two parts of delays can be lumped together as $\tau_k = \tau_{sc} + \tau_{ca}$ for analysis purpose [1]. The control input signal based on the state at instant k will be input to the plant by the actuator at instant $k+\tau_k$ due to transmission delay. Similarly, the number of consecutive data packet dropout d_{sc} and d_{ca} can also be lumped together as $d_k = d_{sc} + d_{ca}$. Assuming that the control input signal calculated from the state at instant k is transmitted to the actuator successfully and the following consecutive d packets are lost, the next control input signal based on time of $k+d+1$ is input to the plant at instant $k+d+1+\tau_{k+d+1}$. During the packet dropout period, the actuator uses the last received data as the input signal.

(iii) The network transmission delay and the number of consecutive data packet dropout are bounded, e.g., $\tau_k \in [0, \ \overline{\tau}]$, $d_k \in [0, \ \overline{d}]$.

According to assumption (i) and (ii), for the NCSs with transmission delay and data packet dropout, when a static state-feedback controller used, the continuous-time model can be described as follows

$$\begin{cases} \dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \boldsymbol{\Delta A}(t))\boldsymbol{x}(t) + (\boldsymbol{B} + \boldsymbol{\Delta B}(t))\boldsymbol{u}(t), \\ \boldsymbol{u}(t) = -\boldsymbol{Kx}(kT), \qquad t \in [kT + \tau_k, (k+1+d)T + \tau_{k+1+d}), \ k = 0, 1, 2, \cdots \end{cases} \tag{4}$$

Set $\eta(t) = t - kT$, yields

$$kT = t - \eta(t) . \tag{5}$$

Replace kT in Eq. (4) with the right side of Eq. (5), we have

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \boldsymbol{\Delta A}(t))\boldsymbol{x}(t) - (\boldsymbol{B} + \boldsymbol{\Delta B}(t))\boldsymbol{Kx}(t - \eta(t)) . \tag{6}$$

According to assumption (iii), $\eta(t)$ in Eq. (6) is also bounded, e.g., $\eta(t) \in [0, \ \overline{\eta}]$ and $\overline{\eta} = \overline{\tau} + (\overline{d}+1)T$. Obviously, $\eta(t)$ is time-varying, piecewise linear continuous.

As a result, the NCSs under assumption (i), (ii) and (iii) can be expressed as Eq. (6), which represents a class of continuous-time dynamic systems with time-varying and bounded input delay. In what follows, we will present new stability and stabilization criteria on the basis of the continuous-time model (6).

3. Stability analysis

At first, two lemmas are given which will be used in this section.

Lemma 1. ([12]): Assume that $\boldsymbol{a} \in \mathrm{R}^{n_a}$, $\boldsymbol{b} \in \mathrm{R}^{n_b}$ and $\boldsymbol{N} \in \mathrm{R}^{n_a \times n_b}$. Then, for any matrices $\boldsymbol{X} \in \mathrm{R}^{n_a \times n_a}$, $\boldsymbol{Y} \in \mathrm{R}^{n_a \times n_b}$ and $\boldsymbol{Z} \in \mathrm{R}^{n_b \times n_b}$ satisfying $\begin{bmatrix} \boldsymbol{X} & \boldsymbol{Y} \\ * & \boldsymbol{Z} \end{bmatrix} \ge 0$, $-2\boldsymbol{a}^{\mathrm{T}}\boldsymbol{Nb} \le \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{X} & \boldsymbol{Y}-\boldsymbol{N} \\ * & \boldsymbol{Z} \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}$ holds.

Lemma 2. ([13]): Given matrices $\boldsymbol{Q}$, $\boldsymbol{H}$ and $\boldsymbol{E}$ with appropriate dimensions, where $\boldsymbol{Q}$ is symmetric, then $\boldsymbol{Q} + \boldsymbol{HF}(t)\boldsymbol{E} + \boldsymbol{E}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{H}^{\mathrm{T}} < 0$ for all matrices $\boldsymbol{F}(t)$ satisfying $\boldsymbol{F}(t)^{\mathrm{T}}\boldsymbol{F}(t) \le \boldsymbol{I}$, if and only if there exists a constant $\varepsilon > 0$, such that $\boldsymbol{Q} + \varepsilon\boldsymbol{HH}^{\mathrm{T}} + \varepsilon^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E} < 0$.

3.1. Nominal system stability

The state equation of the corresponding nominal system of the uncertain NCSs (6) is as follows

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{Ax}(t) - \boldsymbol{BKx}(t - \eta(t)) . \tag{7}$$

The sufficient condition for the stability of the nominal system is given in the following theorem.

Theorem 1. For the given feedback controller gain matrix $\boldsymbol{K}$ and scalar $\overline{\eta} > 0$, if there exist symmetric positive definite matrices $\boldsymbol{P}$, $\boldsymbol{R}$, $\boldsymbol{Z}$, $\boldsymbol{T}$ and matrices $\boldsymbol{G}$, $\boldsymbol{H}$, $\boldsymbol{L_1}$, $\boldsymbol{L}_2$ with appropriate dimensions such that the following inequality (8), (9) and (10) are true, then, when the time-varying delay $0 < \eta(t) \le \bar{\eta}$, the nominal system (7) is asymptotically stable.

$$\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12} & \boldsymbol{\Psi}_{13} & \boldsymbol{\Psi}_{14} \\ * & \boldsymbol{\Psi}_{22} & \boldsymbol{\Psi}_{23} & \boldsymbol{\Psi}_{24} \\ * & * & \boldsymbol{\Psi}_{33} & \boldsymbol{\Psi}_{34} \\ * & * & * & \boldsymbol{\Psi}_{44} \end{bmatrix} < 0, \tag{8}$$

$$\boldsymbol{\Omega}_1 = \begin{bmatrix} \boldsymbol{T} & \boldsymbol{G} \\ * & \boldsymbol{Z} \end{bmatrix} \ge 0, \tag{9}$$

$$\boldsymbol{\Omega}_2 = \begin{bmatrix} \boldsymbol{T} & \boldsymbol{H} \\ * & \boldsymbol{Z} \end{bmatrix} \ge 0, \tag{10}$$

where

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{T}_{11} & \boldsymbol{T}_{12} & \boldsymbol{T}_{13} & \boldsymbol{T}_{14} \\ * & \boldsymbol{T}_{22} & \boldsymbol{T}_{23} & \boldsymbol{T}_{24} \\ * & * & \boldsymbol{T}_{33} & \boldsymbol{T}_{34} \\ * & * & * & \boldsymbol{T}_{44} \end{bmatrix}, \quad \boldsymbol{G} = \begin{bmatrix} \boldsymbol{G}_1 \\ \boldsymbol{G}_2 \\ \boldsymbol{G}_3 \\ \boldsymbol{G}_4 \end{bmatrix}, \quad \boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_1 \\ \boldsymbol{H}_2 \\ \boldsymbol{H}_3 \\ \boldsymbol{H}_4 \end{bmatrix},$$

$\boldsymbol{\Psi}_{11} = \bar{\eta}\boldsymbol{T}_{11} + \boldsymbol{G}_1 + \boldsymbol{G}_1^{\mathrm{T}} + \boldsymbol{R} + \boldsymbol{L}_1^{\mathrm{T}}\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{L}_1$, $\boldsymbol{\Psi}_{12} = \bar{\eta}\boldsymbol{T}_{12} - \boldsymbol{G}_1 + \boldsymbol{H}_1 + \boldsymbol{G}_2^{\mathrm{T}} - \boldsymbol{L}_1^{\mathrm{T}}\boldsymbol{B}\boldsymbol{K}$,
$\boldsymbol{\Psi}_{13} = \bar{\eta}\boldsymbol{T}_{13} - \boldsymbol{H}_1 + \boldsymbol{G}_3^{\mathrm{T}}$, $\boldsymbol{\Psi}_{14} = \bar{\eta}\boldsymbol{T}_{14} + \boldsymbol{G}_4^{\mathrm{T}} + \boldsymbol{P} - \boldsymbol{L}_1^{\mathrm{T}} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{L}_2$,
$\boldsymbol{\Psi}_{22} = \bar{\eta}\boldsymbol{T}_{22} - \boldsymbol{G}_2 - \boldsymbol{G}_2^{\mathrm{T}} + \boldsymbol{H}_2 + \boldsymbol{H}_2^{\mathrm{T}}$, $\boldsymbol{\Psi}_{23} = \bar{\eta}\boldsymbol{T}_{23} - \boldsymbol{H}_2 - \boldsymbol{G}_3^{\mathrm{T}} + \boldsymbol{H}_3^{\mathrm{T}}$,
$\boldsymbol{\Psi}_{24} = \bar{\eta}\boldsymbol{T}_{24} - \boldsymbol{G}_4^{\mathrm{T}} + \boldsymbol{H}_4^{\mathrm{T}} - \boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{L}_2$, $\boldsymbol{\Psi}_{33} = \bar{\eta}\boldsymbol{T}_{33} - \boldsymbol{H}_3 - \boldsymbol{H}_3^{\mathrm{T}} - \boldsymbol{R}$,
$\boldsymbol{\Psi}_{34} = \bar{\eta}\boldsymbol{T}_{34} - \boldsymbol{H}_4^{\mathrm{T}}$, $\boldsymbol{\Psi}_{44} = \bar{\eta}\boldsymbol{T}_{44} + \bar{\eta}\boldsymbol{Z} - \boldsymbol{L}_2 - \boldsymbol{L}_2^{\mathrm{T}}$.

Proof. Using Newton-Leibniz formula, for any matrices $\boldsymbol{M}$ and $\boldsymbol{N}$ with appropriate dimensions, the following equations (11) and (12) are established,

$$2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(\boldsymbol{x}(t) - \boldsymbol{x}(t-\eta(t)) - \textstyle\int_{t-\eta(t)}^{t} \dot{\boldsymbol{x}}(s)\mathrm{d}s) = 0, \tag{11}$$

$$2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{N}(\boldsymbol{x}(t-\eta(t)) - \boldsymbol{x}(t-\bar{\eta}) - \textstyle\int_{t-\bar{\eta}}^{t-\eta(t)} \dot{\boldsymbol{x}}(s)\mathrm{d}s) = 0, \tag{12}$$

where $\boldsymbol{\xi}(t) = \begin{bmatrix} \boldsymbol{x}^{\mathrm{T}}(t) & \boldsymbol{x}^{\mathrm{T}}(t-\eta(t)) & \boldsymbol{x}^{\mathrm{T}}(t-\bar{\eta}) & \dot{\boldsymbol{x}}^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$.

Choose Lyapunov-Krasovskii functional as follows

$$V(t) = V_1(t) + V_2(t) + V_3(t).$$

Here, $V_1(t) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{x}(t)$, $V_2(t) = \int_{t-\bar{\eta}}^{t} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{R}\boldsymbol{x}(s)\mathrm{d}s$, $V_3(t) = \int_{-\bar{\eta}}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta$, where $\boldsymbol{P} > 0$, $\boldsymbol{R} > 0$, $\boldsymbol{Z} > 0$.

Calculating the derivative of $V_1(t)$ and $V_2(t)$ respectively, yields

$$\dot{V}_1(t) = 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{x}}(t), \tag{13}$$

$$\dot{V}_2(t) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{x}(t) - \boldsymbol{x}^{\mathrm{T}}(t-\bar{\eta})\boldsymbol{R}\boldsymbol{x}(t-\bar{\eta}). \tag{14}$$

Calculating the derivative of $V_3(t)$ and adding the left items of Eq. (11) and (12), yields

$$\begin{aligned} \dot{V}_3(t) &= \bar{\eta}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Z}\dot{\boldsymbol{x}}(t) - \textstyle\int_{t-\bar{\eta}}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}\dot{\boldsymbol{x}}(s)\mathrm{d}s \\ &= \bar{\eta}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Z}\dot{\boldsymbol{x}}(t) - \textstyle\int_{t-\eta(t)}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}\dot{\boldsymbol{x}}(s)\mathrm{d}s - \int_{t-\bar{\eta}}^{t-\eta(t)} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}\dot{\boldsymbol{x}}(s)\mathrm{d}s + 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(\boldsymbol{x}(t) - \boldsymbol{x}(t-\eta(t))) \\ &\quad + 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{N}(\boldsymbol{x}(t-\eta(t)) - \boldsymbol{x}(t-\bar{\eta})) - 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}\textstyle\int_{t-\eta(t)}^{t} \dot{\boldsymbol{x}}(s)\mathrm{d}s - 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{N}\int_{t-\bar{\eta}}^{t-\eta(t)} \dot{\boldsymbol{x}}(s)\mathrm{d}s. \end{aligned} \tag{15}$$

According to Lemma 1, when Eq. (9) and (10) in Theorem 1 hold, then

$$-2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}\textstyle\int_{t-\eta(t)}^{t} \dot{\boldsymbol{x}}(s)\mathrm{d}s \le \eta\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{T}\boldsymbol{\xi}(t) + \int_{t-\eta(t)}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}\dot{\boldsymbol{x}}(s)\mathrm{d}s + 2\boldsymbol{\xi}^{\mathrm{T}}(t)(\boldsymbol{G} - \boldsymbol{M})(\boldsymbol{x}(t) - \boldsymbol{x}(t-\eta(t))), \tag{16}$$

$$-2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{N}\textstyle\int_{t-\bar{\eta}}^{t-\eta} \dot{\boldsymbol{x}}(s)\mathrm{d}s \le (\bar{\eta} - \eta)\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{T}\boldsymbol{\xi}(t) + \int_{t-\bar{\eta}}^{t-\eta} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}\dot{\boldsymbol{x}}(s)\mathrm{d}s + 2\boldsymbol{\xi}^{\mathrm{T}}(t)(\boldsymbol{H} - \boldsymbol{N})(\boldsymbol{x}(t-\eta) - \boldsymbol{x}(t-\bar{\eta})) \tag{17}$$

From Eq. (15), (16) and (17), we have

$$\dot{V}_3(t) \le \bar{\eta}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Z}\dot{\boldsymbol{x}}(t) + \bar{\eta}\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{T}\boldsymbol{\xi}(t) + 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{G}(\boldsymbol{x}(t) - \boldsymbol{x}(t-\eta)) + 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{H}(\boldsymbol{x}(t-\eta) - \boldsymbol{x}(t-\bar{\eta})). \tag{18}$$

From Eq. (7), for any matrices $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$ with appropriate dimensions, then

$$2(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{L}_1^{\mathrm{T}} + \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{L}_2^{\mathrm{T}})(\boldsymbol{A}\boldsymbol{x}(t) - \boldsymbol{B}\boldsymbol{K}\boldsymbol{x}(t-\eta(t)) - \dot{\boldsymbol{x}}(t)) = 0\,. \tag{19}$$

According to Eq. (13), (14), (18) and (19), we have

$$\dot{V}(t) \le \boldsymbol{\xi}^{\boldsymbol{T}}(t)\boldsymbol{\Psi}\boldsymbol{\xi}(t)\,.$$

Hence, if Eq. (8), (9) and (10) are established, the inequality $\dot{V}(\boldsymbol{x}(t)) \le -\varepsilon\|\boldsymbol{x}(t)\|^2$ holds, where $\varepsilon = \lambda_{\min}(-\boldsymbol{\Psi}) > 0$ (The symbol of $\lambda_{\min}(\bullet)$ denotes the smallest eigenvalue of the corresponding positive definite matrix). According to Lyapunov-Krasovskii theorem, the nominal system (7) is asymptotically stable and Theorem 1 is proved.

The term $-\int_{t-\bar{\eta}}^{t-\eta}\dot{x}^{\mathrm{T}}(s)\boldsymbol{Z}\dot{x}(s)\mathrm{d}s$ from $\dot{V}_3(t)$ is ignored in most of the similar literatures like [8-9] when analyzing the stability of the NCSs with time-varying delay and packet dropout. Thought it was firstly put forward that the ignorance of this term would lead to conservativeness in literature [11], the method used in that paper was not very reasonable because it simply expanded two cross product items about $\eta(t)$ and $\bar{\eta} - \eta(t)$ to $\bar{\eta}$, which could also lead to conservativeness. In this paper we use a more effective approach through Moon's inequality and obtain better results with smaller conservativeness.

3.2. Parameter uncertain system stability

Theorem 2. For the given feedback controller gain matrix $\boldsymbol{K}$ and scalar $\bar{\eta} > 0$, if there exist symmetric positive definite matrices $\hat{\boldsymbol{P}}$, $\hat{\boldsymbol{R}}$, $\hat{\boldsymbol{Z}}$, $\hat{\boldsymbol{T}}$ and matrices $\hat{\boldsymbol{G}}$, $\hat{\boldsymbol{H}}$, $\hat{\boldsymbol{L}}_1$, $\hat{\boldsymbol{L}}_2$ with appropriate dimensions such that the following inequality (20), (21) and (22) are established, then, when the time-varying delay $0 < \eta(t) \le \bar{\eta}$, the NCSs (6) with parameter uncertainties is asymptotically stable.

$$\hat{\boldsymbol{\Psi}} = \begin{bmatrix} \hat{\boldsymbol{\Psi}}_{11} & \hat{\boldsymbol{\Psi}}_{12} & \hat{\boldsymbol{\Psi}}_{13} & \hat{\boldsymbol{\Psi}}_{14} & \hat{\boldsymbol{L}}_1^{\mathrm{T}}\boldsymbol{D} & \boldsymbol{E}_a^{\mathrm{T}} \\ * & \hat{\boldsymbol{\Psi}}_{22} & \hat{\boldsymbol{\Psi}}_{23} & \hat{\boldsymbol{\Psi}}_{24} & 0 & -\boldsymbol{K}^{\mathrm{T}}\boldsymbol{E}_b^{\mathrm{T}} \\ * & * & \hat{\boldsymbol{\Psi}}_{33} & \hat{\boldsymbol{\Psi}}_{34} & 0 & 0 \\ * & * & * & \hat{\boldsymbol{\Psi}}_{44} & \hat{\boldsymbol{L}}_2^{\mathrm{T}}\boldsymbol{D} & 0 \\ * & * & * & * & -\boldsymbol{I} & 0 \\ * & * & * & * & * & -\boldsymbol{I} \end{bmatrix} < 0\,, \tag{20}$$

$$\hat{\boldsymbol{\Omega}}_1 = \begin{bmatrix} \hat{\boldsymbol{T}} & \hat{\boldsymbol{G}} \\ * & \hat{\boldsymbol{Z}} \end{bmatrix} \ge 0\,, \tag{21}$$

$$\hat{\boldsymbol{\Omega}}_2 = \begin{bmatrix} \hat{\boldsymbol{T}} & \hat{\boldsymbol{H}} \\ * & \hat{\boldsymbol{Z}} \end{bmatrix} \ge 0\,, \tag{22}$$

where $\hat{\boldsymbol{\Psi}}_{ij} = \varepsilon\boldsymbol{\Psi}_{ij}(i,j = 1,2,3,4)$ and $\boldsymbol{\Psi}_{ij}$ is the same as that defined in Theorem 1.

Proof. Similar to literature [13], use $\boldsymbol{A} + \boldsymbol{D}\boldsymbol{F}(t)\boldsymbol{E}_a$ and $\boldsymbol{B} + \boldsymbol{D}\boldsymbol{F}(t)\boldsymbol{E}_b$ to replace $\boldsymbol{A}$ and $\boldsymbol{B}$ in Eq. (8) respectively, $\varepsilon > 0$ is multiplied on both sides of the equation and then use Lemma 2, we have

$$\varepsilon\boldsymbol{\Psi} + \varepsilon^2\begin{bmatrix} \boldsymbol{L}_1^{\mathrm{T}}\boldsymbol{D} \\ 0 \\ 0 \\ \boldsymbol{L}_2^{\mathrm{T}}\boldsymbol{D} \end{bmatrix}\boldsymbol{F}(t)\begin{bmatrix} \boldsymbol{E}_a & -\boldsymbol{E}_b\boldsymbol{K} & 0 & 0 \end{bmatrix} + \begin{bmatrix} \boldsymbol{E}_a^{\mathrm{T}} \\ -\boldsymbol{K}^{\mathrm{T}}\boldsymbol{E}_b^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}\boldsymbol{F}^{\mathrm{T}}(t)\begin{bmatrix} \boldsymbol{D}^{\mathrm{T}}\boldsymbol{L}_1 & 0 & 0 & \boldsymbol{D}^{\mathrm{T}}\boldsymbol{L}_2 \end{bmatrix} < 0\,. \tag{23}$$

Set $\hat{\boldsymbol{P}} = \varepsilon\boldsymbol{P}$, $\hat{\boldsymbol{R}} = \varepsilon\boldsymbol{R}$, $\hat{\boldsymbol{Z}} = \varepsilon\boldsymbol{Z}$, $\hat{\boldsymbol{T}} = \varepsilon\boldsymbol{T}$, $\hat{\boldsymbol{G}} = \varepsilon\boldsymbol{G}$, $\hat{\boldsymbol{H}} = \varepsilon\boldsymbol{H}$, $\hat{\boldsymbol{L}}_1 = \varepsilon\boldsymbol{L}_1$, $\hat{\boldsymbol{L}}_2 = \varepsilon\boldsymbol{L}_2$ and Eq. (23) is equivalent to Eq. (20) according to Schur complement lemma. This completes the proof.

4. Simulation examples

Example 1. Consider the stability of the following continuous-time system [1].

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}u(t)\,. \tag{24}$$

This example has been discussed in some similar literatures [1, 8, 14, 15] and the maximum allowable delay bounds (MADB [14]), that guarantees the stability of the NCSs, is used to measure the conservativeness. The MADB is greater, the smaller the conservativeness. The greatest MADB in those papers is 0.8871 s from [15]. In this example we take the same feedback gain $\boldsymbol{K}=[3.75 \quad 11.5]$ as in literature [1, 8, 14, 15] and obtain the MADB=1.0432 s by Theorem 1. It is clear that our result is less conservative.

Assuming the initial state $\boldsymbol{x}_0=[-5 \quad 10]^{\mathrm{T}}$, when the network transmission delay is 0.1432 s, sampling period is 0.3 s and maximum number of consecutive packet dropout is 2, e.g., the corresponding MADB=1.0432 s, use MATLAB/TrueTime to simulate the NCSs and the corresponding state response curves are shown in Fig. 2. The stability is apparent under the above specific condition from Fig. 2.

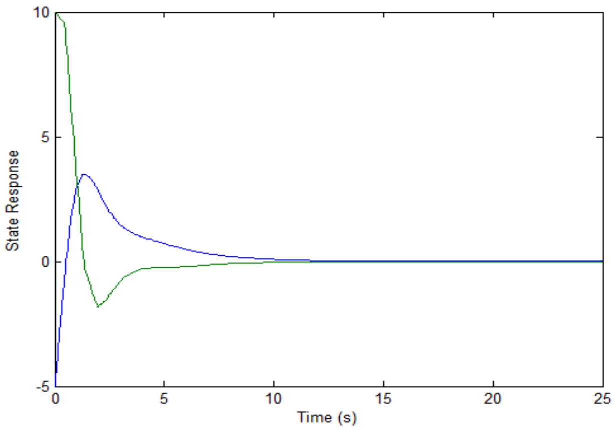

Fig. 2 State response curves of the NCSs

Example 2. Consider the robust stability of the uncertain system (6) with the following parameter

$$\boldsymbol{A}=\begin{bmatrix}-0.5 & -2\\ 1 & -1\end{bmatrix},\ \boldsymbol{B}=\begin{bmatrix}0.5 & 1\\ 0 & -0.6\end{bmatrix},\ \boldsymbol{K}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\boldsymbol{E}_a=-\boldsymbol{E}_b=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix},\ \boldsymbol{D}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

This example has also been discussed in some similar literatures [11, 13]. The best result is MADB=0.3365 s from [11]. Here we use Theorem 2 and obtain MADB=0.3972 s. The result shows that our criteria can also improve the conservativeness of the uncertain systems.

5. Conclusion

This paper researches the stability of the networked control systems with parameter uncertainties. The NCSs with time-delay and data packet dropout are modeled as a class of uncertain continuous-time systems with time-varying input delay. Using the free weighting matrix approach, the LMI-based sufficient conditions for asymptotic stability of the nominal systems and uncertain systems are derived. We use more reasonable method to transform the integral term $-\int_{t-\bar{\eta}}^{t-\eta}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}\dot{\boldsymbol{x}}(s)\mathrm{d}s$, which is neglected in most of the similar literatures, so less conservative results are obtained. Simulation results show that the performance of the system has been improved obviously. Through the approaches presented in literature [7], the stability criteria can also be transformed into stabilization algorithm, which is omitted due to space limit.

References

[1] Zhang W, Branicky MS, Phillips SM. Stability of networked control systems. IEEE Control System Magazine, 2001, 21(1): 84-99.

[2] Li HB, Sun ZQ, Sun FC. Networked control systems: an overview of state-of-the-art and the prospect in future research. Control Theory & Applications, 2010, 27(2): 238-243.

[3] Hu SS, Zhu QX. Stochastic optimal control and analysis of stability of networked control systems with long delay. Automatica, 2003, 39(11): 1877-1884.

[4] Yu M, Wang L, Chu T. Stabilization of networked control systems with data packet dropout and networked delays via switching system approach. Proceedings of the 43rd IEEE Conference on Decision and Controls, Atlantis, Paradise Island, Bahamas: IEEE Press, 2004: 3539-3544.

[5] Guo YF, Li SY. A new networked predictive control approach for systems with random network delay in the forward channel. International Journal of Systems Science, 2010, 41(5): 511–520.

[6] Zhang HG, Li M, Yang J. Fuzzy model-based robust networked control for a class of nonlinear systems. IEEE Transactions on Systems, Man, and Cybernetics, 2009, 39(2): 437-447.

[7] Fridman E, Seuret A, Richard JP. Robust sampled-data stabilization of linear systems: an input delay approach. Automatic, 2004, 40(9): 1441-1446.

[8] Yue D, Han QL, Peng C. State feedback controller design of networked control systems. IEEE Transactions on Circuits and Systems – II: Express Briefs, 2004, 51(11): 640-644.

[9] Guo YF, Li SY. H-infinity state feedback controller design for networked control systems. Control Theory & Applications, 2008, 25(3): 825-835.

[10] Moon YS, Park P, Kwon WH. Robust stabilization of uncertain input-delayed systems using reduction method. Automatic, 2001, 37(2): 307-312.

[11] He Y, Wang QG, Xie LH. Further improvement of free-weighting matrices technique for systems with time-varying delay. IEEE Transactions on Automatic Control, 2007, 52(2): 293-299.

[12] Kim DS, Lee YS, Kwon WH. Maximum allowable delay bounds of networked control systems. Control Engineering Practice, 2003, 11(11): 1301-1313.

[13] Wu M, He Y, She JH. Delay-dependent criteria for robust stability of time-varying delay systems. Automatic, 2004, 40(8): 1435-1439.

[14] Park HS, Kim YS, Kim DS. A scheduling method for network-based control systems. IEEE Transactions on Control Systems Technology, 2002, 10(3): 318-330.

[15] Yue D, Han QL, Lam J. Network-based robust H_∞ control of systems with uncertainty. Automatica, 2005, 41(2): 307-312.